

QUADRATIC EQUATIONS WORD PROBLEMS

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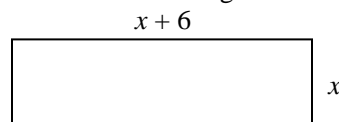
Sample Problems

1. The length of a rectangle is 6 inches more than its width. The area of the rectangle is 91 square inches. Find the dimensions of the rectangle.

The formula for the **area** of a rectangle is: **Length X Width = Area**
 $LW = A$

Step 1 - Draw the picture of the rectangle

Since the length is 6 more than the width we let the width = x and the length = $x + 6$



Step 2 - Write the equation using the formula $LW = A$

$$x(x + 6) = 91$$

Step 3 - Solve the equation

$$\begin{aligned} x^2 + 6x &= 91 \\ x^2 + 6x - 91 &= 0 \\ (x - 7)(x + 13) &= 0 \end{aligned}$$

$$\begin{array}{l|l} x - 7 = 0 & x + 13 = 0 \\ x = 7 & x = -13 \\ & \text{(This not a valid answer for} \\ & \text{the side of a rectangle.)} \end{array}$$

The **length** is 13 and the **width** is 7

2. The product of two consecutive odd integers is 1 less than four times their sum. Find the two integers. We recall from Elementary Algebra that that we write two consecutive odd integers as:

Let: $n = 1\text{st number}$
 $n + 2 = 2\text{nd number}$

Step 1 - Write the equation

$$n(n + 2) = 4[n + (n + 2)] - 1$$

The **PRODUCT** of the two numbers Four times their **SUM** Their **SUM** One less than 4 times their sum

Step 2 - Solve the equation

$$\begin{aligned} n^2 + 2n &= 4[2n + 2] - 1 \\ n^2 + 2n &= 8n + 8 - 1 \\ n^2 + 2n &= 8n + 7 \\ n^2 - 6n - 7 &= 0 \\ (n - 7)(n + 1) &= 0 \end{aligned}$$

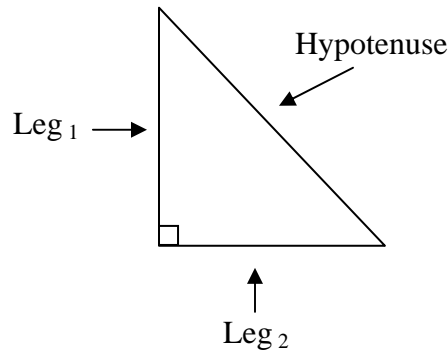
We see to the right that we get **TWO** sets of answers:
7 and 9
-1 and 1



The two numbers: 7 and 9	The two numbers: -1 and 1
$7 \times 9 = 4[7 + (7 + 2)] - 1$	$-1 \times 1 = 4[-1 + (-1 + 2)] - 1$
$63 = 4[7 + 9] - 1$	$-1 = 4[-1 + 1] - 1$
$63 = 4(16) - 1$	$-1 = 4(0) - 1$
$63 = 64 - 1$	$-1 = 0 - 1$
$63 = 63$	$-1 = -1$

3. The hypotenuse of a right triangle is 6 more than the shorter leg. The longer leg is three more than the shorter leg. Find the length of the shorter leg.

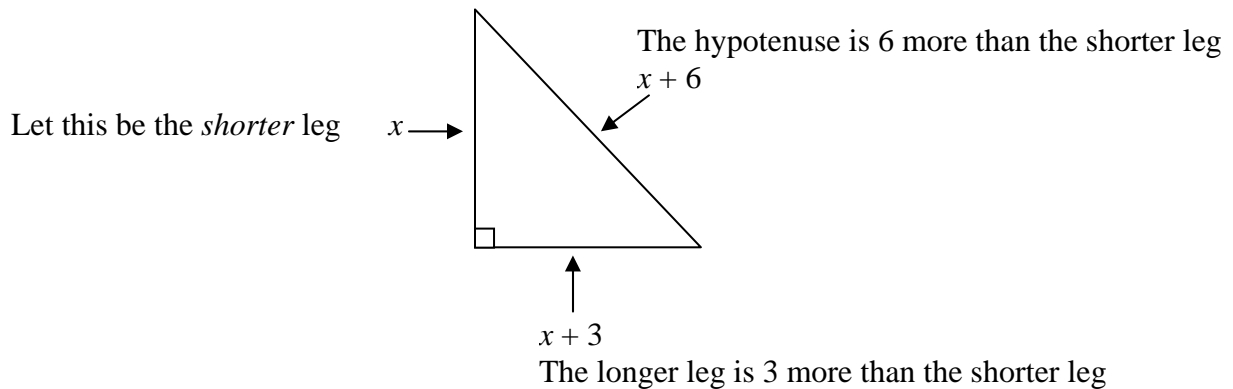
Here we use the **Pythagorean Theorem** which states that in a **right triangle**:
The **sum** of the **squares** of the **legs** is equal to the **square** of the **hypotenuse**.



$$(\text{Leg}_1)^2 + (\text{Leg}_2)^2 = (\text{Hypotenuse})^2$$

** Note: It does not matter which leg we use for Leg₁ or Leg₂

To work out the problem we can define the sides of the triangle according to the figure below:



Step 1 - Write the equation

$$x^2 + (x + 3)^2 = (x + 6)^2$$

Step 2 - Solve the equation

By using the **SQUARE OF A BINOMIAL FORMULA**

$$x^2 + x^2 + 6x + 9 = x^2 + 12x + 36$$

$$2x^2 + 6x + 9 = x^2 + 12x + 36$$

$$x^2 - 6x - 27 = 0$$

$$(x - 9)(x + 3) = 0$$

$$x - 9 = 0 \quad | \quad x + 3 = 0$$

$$x = 9 \quad | \quad x = -3$$

The **shorter leg is 9**

(This not a valid answer since we cannot have the side of a triangle being a negative value.)

Solve the following WORD PROBLEMS:

1. The length of a rectangle is 2 times its width. The area of the rectangle is 72 square inches. Find the dimensions of the rectangle.
2. The length of a rectangle is 4 times its width. The area of the rectangle is 144 square inches. Find the dimensions of the rectangle.
3. The length of a rectangular garden is 4 yards more than its width. The area of the garden is 60 square yards. Find the dimensions of the garden.
4. The width of a rectangle is 11 inches less than its length. Find the dimensions of the rectangle if the area is 80 square inches.
5. The length of a rectangle exceeds its width by 3 inches. The area of the rectangle is 70 square inches, find its dimensions.
6. The length of a rectangle is 3 centimeters more than the width. The area is 108 square centimeters. Find the length and width of the rectangle.
7. The width of a rectangle is 5 meters less than its length. The area is 84 square meters. Find the dimensions of the rectangle.
8. The length of a rectangle is twice the width. The area is 50 square inches. Find the dimensions of the rectangle.
9. The length of a rectangle is 1 foot more than twice the width. The area is 55 square feet. Find the dimensions of the rectangle.
10. The length of a rectangle is 2 less than three times the width. Find the dimensions of the rectangle if the area is 65 square meters.
11. The length of a rectangle is 7 meters less than twice the width. Find the dimensions if the area is 60 square meters.
12. The product of two consecutive integers is 56. Find the integers.
13. The product of two consecutive odd integers is 99. Find the integers.
14. Find two consecutive even integers such that the square of the smaller is 10 more than the larger.
15. The product of two consecutive odd integers is 1 less than twice their sum. Find the integers.
16. The product of two consecutive integers is three less than three times their sum. Find the integers.
17. The product of two consecutive even integers is 6 more than three times their sum. Find the integers.
18. The product of two consecutive odd integers is 77 more than twice the larger. Find the integers.
19. The product of two consecutive integers is 5 more than three times the larger. Find the integers.
20. Find three consecutive integers such that four times the sum of all three is 2 times the product of the larger two.
21. Find three consecutive integers such that three times the sum of all three equals the product of the larger two.
22. The medium side of a right triangle is 7 more than the shortest side. The longest side is 7 less than 3 times the shortest side. Find the length of the shortest side of the triangle.
23. One leg of a right triangle is one inch shorter than the other leg. If the hypotenuse is 5 inches, find the length of the shorter leg.

24. The longer leg of a right triangle is two inches more than twice the length of the shorter leg. The hypotenuse is two inches less than three times the length of the shorter leg. Find the length of the *hypotenuse*.
25. The longer leg of a right triangle is ten less than three times the shorter leg. The hypotenuse is 4 more than the shorter leg. Find the length of the shorter leg.
26. The hypotenuse of a right triangle is 3 less than twice the shorter leg. The length of the other leg is 3 more than the shorter leg. Find the length of the shorter leg.
27. The hypotenuse of a right triangle is 1 centimeter longer than the longer leg. The shorter leg is 7 centimeters shorter than the longer leg. Find the length of the longer leg of the triangle.
28. The longer leg of a right triangle is 1 meter longer than the shorter leg. The hypotenuse is 1 meter shorter than twice the shorter leg. Find the length of the shorter leg of the triangle.
29. A ladder is resting against a wall. The top of the ladder touches the wall at a height of 15 feet. Find the distance from the wall to the bottom of the ladder if the length of the ladder is one foot more than twice its distance from the wall.
30. Two cars leave an intersection. One car travels north; the other travels east. When the car traveling north had gone 24 miles, the distance between the cars was four miles more than three times the distance traveled by the car heading east. Find the distance between the cars at that time.

ANSWERS

1. $W = 6, L = 12$	16. 0, 1 & 5, 6
2. $W = 6, L = 24$	17. $-2, 0$ & 6, 8
3. $W = 6, L = 10$	18. $-9, -7$ & 9, 11
4. $W = 5, L = 16$	19. 4, 5 & $-2, -1$
5. $W = 7, L = 10$	20. 4, 5, 6 & $-1, 0, 1$
6. $W = 9, L = 12$	21. 7, 8, 9 & $-1, 0, 1$
7. $W = 7, L = 12$	22. 8
8. $W = 5, L = 10$	23. 3 inches
9. $W = 5, L = 11$	24. 13 inches
10. $W = 5, L = 13$	25. 6
11. $W = \frac{15}{2}, L = 8$	26. 9
12. 7, 8 & $-8, -7$	27. 12
13. 9, 11 & $-11, -9$	28. 3
14. 4 & 6	29. 8 feet
15. 3, 5 & $-1, 1$	30. 25 miles