SUPPLEMENTARY SHEET 7

SQUARE ROOTS

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SIMPLIFYING SQUARE ROOTS:

Remember: When we *simplify* a square root we look for the *largest* perfect square which is a factor of the given number.

Example:

Simplify $\sqrt{48}$

Even though **4** is a perfect square that is a factor or 48, the *largest* perfect square which is a factor if 48 is **16**. If we used **4** we would have to simplify *twice* instead of only *once.*

Using **4** we would have:

$$\sqrt{48} = \sqrt{4} \bullet \sqrt{12} = 2\sqrt{12} = 2\sqrt{4} \bullet \sqrt{3} = 2 \bullet 2\sqrt{3} = 4\sqrt{3}$$

THIS IS MUCH TOO MUCH WORK!!!

Here is the *better* way: $\sqrt{48} = \sqrt{16} \bullet \sqrt{3} = 4\sqrt{3}$

Simplify the following square roots:

1. $\sqrt{8}$
2. $\sqrt{20}$
3. $\sqrt{45}$
4. $\sqrt{72}$
5. $-\sqrt{108}$
6. $-\sqrt{128}$
7. $-\sqrt{300}$
8. $\sqrt{-12}$
9. $-\sqrt{50}$

Answers: 1. $2\sqrt{2}$ 2. $2\sqrt{5}$ 3. $3\sqrt{5}$ 4. $6\sqrt{2}$ 5. $-6\sqrt{3}$ 6. $-8\sqrt{2}$ 7. $-10\sqrt{3}$ 8. not a real number 9. $-5\sqrt{2}$

MULTIPLYING and SIMPLIFYING SQUARE ROOTS

Example:

Multiply and Simplify: $2\sqrt{54} \bullet 5\sqrt{75}$

This can be done in two different ways:

THE HARDER WAY!!

$2\sqrt{54} \bullet 5\sqrt{75} = 2 \bullet 5\sqrt{54 \bullet 75}$

$= 10\sqrt{4050}$

Now you are left with trying to find perfect squares that are factors of this number.

This can be difficult.

We know **9** goes into 4050 by the divisibility rule for **9** (**9** goes into a number if the sum of the digits of the number is divisible by **9**). The sum of 4+0+5+0 is 9. Since 9 is divisible by **9**, **9** goes into 4050.

Therefore: $10\sqrt{4050} = 10\sqrt{9 \bullet 450}$

$= 10 \bullet 3\sqrt{450}$

$= 30\sqrt{9 \bullet 50}$

$= 30 \bullet 3\sqrt{25 \bullet 2}$

$= 90 \bullet 5\sqrt{2}$

$= 450\sqrt{2}$

THE EASIER WAY!!

First we simplify each square root:

$2\sqrt{54} \bullet 5\sqrt{75} = 2\sqrt{9 \bullet 6} \bullet 5\sqrt{25 \bullet 3}$

Take out each perfect square.

$= 2 \bullet 3 \bullet 5 \bullet 5\sqrt{6 \bullet 3}$

Factor 6 into $2 \bullet 3$

$= 150\sqrt{2 \bullet 3 \bullet 3}$

We now have another perfect square created by the $3 \bullet 3$

$= 150 \bullet 3\sqrt{2}$

Multiply $150 \bullet 3$

$= 450\sqrt{2}$

Multiply and Simplify:

1. $\sqrt{24} \bullet \sqrt{3}$
2. $\sqrt{3} \bullet \sqrt{21}$
3. $\sqrt{6} \bullet \sqrt{12}$
4. $\sqrt{50} \bullet \sqrt{9}$
5. $\sqrt{12} \bullet \sqrt{8}$
6. $\sqrt{75} \bullet \sqrt{6}$
7. $\sqrt{6} \bullet \sqrt{18}$
8. $\sqrt{50} \bullet \sqrt{10}$
9. $\sqrt{26} \bullet \sqrt{13}$

Answers: 1. $6\sqrt{2}$ 2. $3\sqrt{7}$ 3. $6\sqrt{2}$ 4. $15\sqrt{2}$ 5. $4\sqrt{6}$ 6. $15\sqrt{2}$ 7. $6\sqrt{3}$ 8. $10\sqrt{5}$ 9. $13\sqrt{2}$

ADDING and SUBTRACTING SQUARE ROOTS

Just as we must have *like terms* when adding or subtracting algebraic expressions we must have *like square roots* when we add or subtract square roots.

Example: Find the answer to $4\sqrt{12} + 5\sqrt{3} - 2\sqrt{75}$

The square roots in this problem are not *like* square roots.

First we need to simplify each square root.

$$4\sqrt{12} + 5\sqrt{3} - 2\sqrt{75} = 4\sqrt{4 \bullet 3} + 5\sqrt{3} - 2\sqrt{25 \bullet 3}$$

$$= 4 \bullet 2\sqrt{3} + 5\sqrt{3} - 2 \bullet 5\sqrt{3}$$

$$= 8\sqrt{3} + 5\sqrt{3} - 10\sqrt{3} = 3\sqrt{3}$$

Find the answers to each of the following:

1. $5\sqrt{3} + 2\sqrt{3} + 8\sqrt{3}$
2. $5\sqrt{3} + \sqrt{3} - 2\sqrt{3}$
3. $\sqrt{2} + \sqrt{50}$
4. $\sqrt{27} + \sqrt{75}$
5. $\sqrt{5} + \sqrt{45} + \sqrt{80}$
6. $\sqrt{72} - \sqrt{50}$
7. $\sqrt{12} - \sqrt{48} + \sqrt{3}$
8. $3\sqrt{2} + 2\sqrt{32}$
9. $5\sqrt{27} - \sqrt{108} + 2\sqrt{75}$
10. $3\sqrt{40} - \sqrt{90}$
11. $3\sqrt{8} - \sqrt{2}$
12. $5\sqrt{8} - 3\sqrt{18} + \sqrt{3}$
13. $3\sqrt{50} - 5\sqrt{18}$
14. $3\sqrt{28} - 2\sqrt{63}$
15. $\sqrt{98} - 4\sqrt{8} + 3\sqrt{128}$
16. $\frac{1}{2}\sqrt{20} + \sqrt{45}$
17. $\frac{2}{3}\sqrt{18} - \sqrt{72}$
18. $4\sqrt{18} - \frac{3}{4}\sqrt{32} - \frac{1}{2}\sqrt{8}$

Answers:	1. $15\sqrt{3}$	2. $4\sqrt{3}$	3. $6\sqrt{2}$	4. $8\sqrt{3}$	5. $8\sqrt{5}$	6. $\sqrt{2}$
	7. $-\sqrt{3}$	8. $11\sqrt{2}$	9. $19\sqrt{3}$	10. $3\sqrt{10}$	11. $5\sqrt{2}$	12. $\sqrt{2} + \sqrt{3}$
	13. 0	14. 0	15. $23\sqrt{2}$	16. $4\sqrt{5}$	17. $-4\sqrt{2}$	18. $8\sqrt{2}$

SOLVING QUADRATIC EQUATIONS USING THE SQUARE ROOT PROPERTY

Example 1:

Solve: $x^2 = 25$

The way we are used to solving this equation is to set the equation equal to *zero*.

$$x^2 - 25 = 0$$

Next we factor:

$(x - 5)$	$(x + 5) = 0$
$x - 5 = 0$	$x + 5 = 0$
$x = 5$	$x = -5$

The Solution Set is $\{-5, 5\}$.

Another way we can write this is $\{\pm 5\}$

When we have an equation involving the difference of squares (i.e. $x^2 - a^2 = 0$) we should realize that we will always get an answer that is always *plus* and *minus* the *square root* of a^2

In this case the answer is $-\sqrt{25}$ or $+\sqrt{25}$ or -5 and $+5$

This fact can be used to solve the equation

$$x^2 = 25$$

by using the **SQUARE ROOT PROPERTY**.

> The **SQUARE ROOT PROPERTY** states that if we have an equation of the form:
>
> $$x^2 = a$$
>
> then $x = \sqrt{a}$ or $x = -\sqrt{a}$

so in solving $x^2 = 25$ we have:

$$x = -\sqrt{25} \text{ or } +\sqrt{25}$$

which we can write $x = \pm\sqrt{25} = \pm 5$

In Solution Set form: $\{\pm 5\}$

Example 2:

We can now use the **SQUARE ROOT PROPERTY** to solve equations that we could not solve before.

Solve: $x^2 = 19$

If we set this equal to *zero* we can see that this is *not* a difference of squares, however, by the **SQUARE ROOT PROPERTY:**

$$x = \sqrt{19} \text{ or } x = -\sqrt{19} \text{ or}$$
$$x = \pm\sqrt{19}$$

In Solution Set form $\{\pm\sqrt{19}\}$

* *

Example 3:

Solve: $x^2 = 20$

We now know that $x = \sqrt{20}$ or $x = -\sqrt{20}$ or

Simplifying: $x = \sqrt{4 \bullet 5} = 2\sqrt{5}$ or

$x = -\sqrt{4 \bullet 5} = -2\sqrt{5}$

so $x = \pm 2\sqrt{5}$

In Solution Set form $\{\pm 2\sqrt{5}\}$

Solve each of the following equations. Express radicals in simplest form.

1. $x^2 = 64$	2. $z^2 = 100$	3. $y^2 = 23$
4. $x^2 = 5$	5. $a^2 = 72$	6. $m^2 = 48$
7. $p^2 = 17$	8. $x^2 = 24$	9. $b^2 = 108$
10. $x^2 = 75$	11. $y^2 = 121$	12. $x^2 = 35$

Answers: 1. $\{\pm 8\}$ 2. $\{\pm 10\}$ 3. $\{\pm\sqrt{23}\}$ 4. $\{\pm\sqrt{5}\}$ 5. $\{\pm 6\sqrt{2}\}$ 6. $\{\pm 4\sqrt{3}\}$ 7. $\{\pm\sqrt{17}\}$ 8. $\{\pm 2\sqrt{6}\}$ 9. $\{\pm 6\sqrt{3}\}$ 10. $\{\pm 5\sqrt{3}\}$ 11. $\{\pm 11\}$ 12. $\{\pm\sqrt{35}\}$